

MANNED RETURN FROM SPACE

By R. B. HILDEBRAND

Chief, Advanced Systems Research, Aero-Space Division, Boeing Airplane Company,
Seattle, Washington*

Abstract—This paper presents an analysis and study of earth entry of manned vehicles at speeds greater than 26,000 fps, with particular emphasis on the lunar return mission. Ballistic entries are compared with lifting entries. The effect of lift is shown to reduce entry accelerations and provide a wider permissible guidance tolerance for single pass entries. When the total heat load increases moderately, lift and drag modulation widen the entry corridor significantly. A high maximum lift coefficient will also widen the entry corridor, and minimize the total heat load.

INTRODUCTION

MAN's conquest of space is imminent. Within the next years will come manned earth orbital flights. Soon after that, man will venture farther into space, first to the moon and eventually to the planets. Before such a voyage can begin, a way of returning man and materials safely to the earth must be developed.

TYPES OF RETURN

As an example of such space voyages, let us examine the lunar return mission. Two extremes of trajectories may be considered: the direct return, and the retrograde return. In the direct return, the vehicle approaches the earth in the same direction as the earth's rotation. In the retrograde return, the vehicle approaches the earth in the direction opposite to the earth's rotation. See Fig. 1.

The following table summarizes the characteristics of two typical trajectories representative of these types.

The minimum velocity of these trajectories during midcourse flight is 5000 fps. The guidance tolerance at lunar launch is dependent upon the flight path angle tolerance at earth entry, but is probably more stringent than that required for ICBM's if no midcourse corrections are made.

* *Acknowledgement*—The author wishes to express his gratitude to Robert H. Smith, Joseph Menard, Kenneth Peterson, Lowell B. Eldrenkamp, and Robert Roe for their efforts, contributions and stimulating discussions which made this paper possible.

RETURN FROM THE MOON

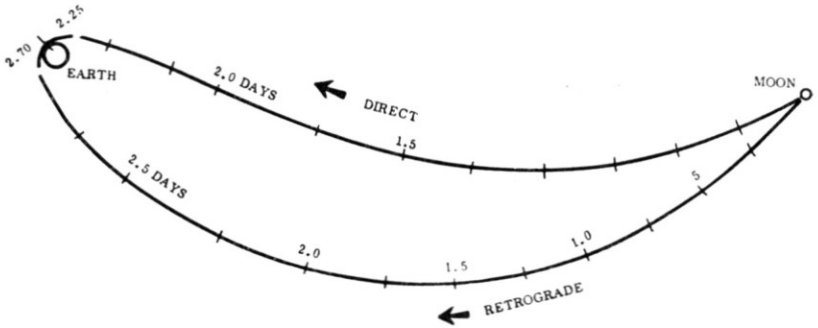


FIG. 1.

MIDCOURSE CORRECTIONS

2 1/4-DAY LUNAR RETURN

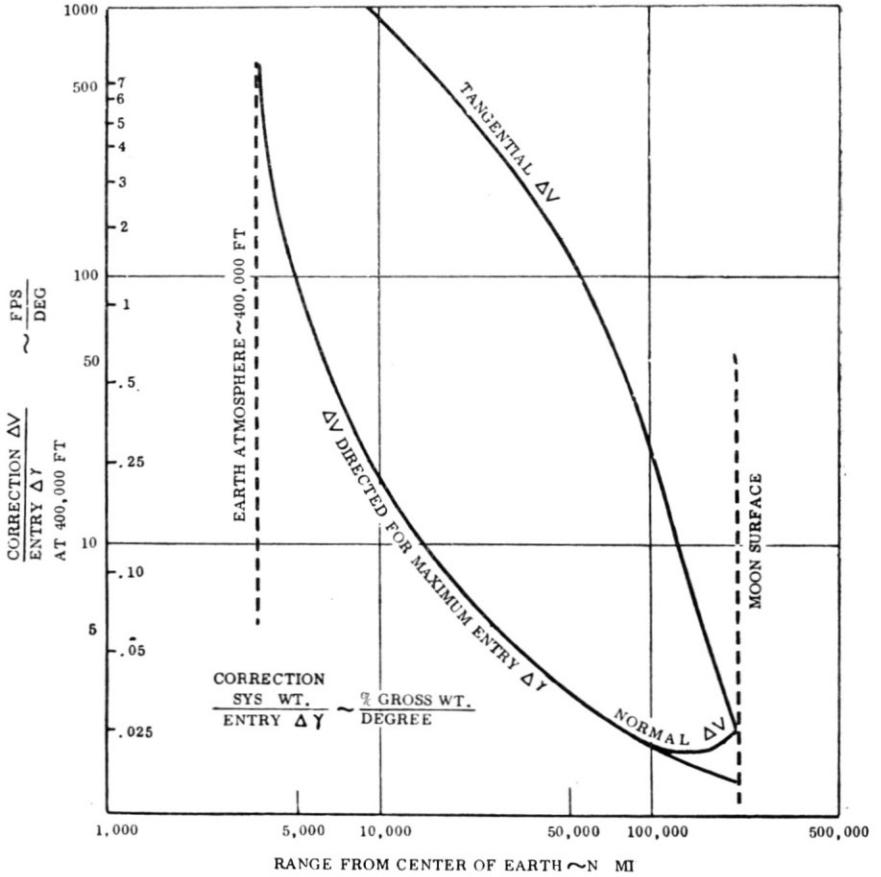


FIG. 2.

TABLE 1

		Lunar Launch		
		Velocity (fps)	Angle γ	Flight Time (Days)
Direct Return		9000	39°	2.25
Retrograde Return		9000	24°	2.70

		Earth Re-Entry			
		Entry Angle		*	
	Angle γ	Altitude (ft.)	Sensitivity to Launch Vel. (°/fps)	Inertial Velocity (fps)	Relative Velocity (fps)
Direct Return	6°	400,000	0.8	36,200	34,600
Retrograde Return	6°	400,000	1.2	36,200	37,740

* These velocities are relative to the center of the earth-moon system. Relative to fixed earth, $V_i = 36,260$ Direct, 36,140 Retrograde

As the space vehicle journeys toward earth, its speed first decreases and then increases. When the vehicle nears the earth, course corrections require greater and greater rocket power (although they are probably more accurate closer to the earth). See Fig. 2. Typical vehicle weight increases for this rocket power for a 1° entry angle change are indicated on the ordinate. A specific impulse of 300 seconds and a propellant loading fraction of 0.80 were assumed.

ENTRY TECHNIQUES

Three basic techniques are available for decelerating the space vehicle:

1. Rocket
2. Atmospheric drag—multiple pass
3. Atmospheric drag—single pass

Some combinations of techniques may be desirable. In ref.⁽¹⁾ it was shown that the single pass entry is the more likely choice. In the first two techniques, there are either large weight increases (rocket), or the re-entry time is extremely sensitive to the entry flight path angle (multiple pass).

For single pass entries, the vehicle re-enters and remains in the earth's atmosphere using aerodynamic drag for deceleration. To successfully re-enter in a single pass, the space vehicle must be within the so-called re-entry corridor, which has two bounds: the upper bound, the overshoot or skip limit; and the lower bound, the aerodynamic heating and/or acceleration limit. This corridor then represents the target for the guidance and control system.

MOTION ANALYSIS

To analyze entry trajectories, a number of investigators have used the fact that the density variation of the atmosphere is nearly an exponential function^(2,3,4,5,6). Assuming that the atmosphere density can be represented by an exponential function, the equations of motion become amenable to analysis without resorting to numerical methods for solution. The scale height is usually assumed to be about 23,000 feet for the earth's atmosphere. An analysis was made of the scale height of the 1959 ARDC Model Atmosphere⁽⁷⁾. This analysis shows that in the region of interest, between 150,000 feet and 270,000 feet, the scale height varies between 17,000 and 30,000 feet.

To obtain the results contained in this paper, entry trajectories were calculated using a high-speed digital computing machine (IBM 704). The complete equations of motion-of-a-point mass and the 1959 ARDC Model Atmosphere were used. The equations of motion are:

$$\ddot{r} - r\dot{\theta}^2 - r(\dot{\phi} + \omega)^2 \sin^2 \theta + \frac{\rho C_D S}{2m} V_r \dot{r} - \frac{\rho C_L S}{2m} V_r r V^* + GM \left[\frac{1}{r^2} + \frac{3Jr_e^2}{r^4} \left(\frac{1}{3} - \cos^2 \theta \right) \right] = 0 \quad (1)$$

$$\frac{d}{dt} [r^2(\dot{\phi} + \omega) \sin^2 \theta] + \frac{\rho C_D S}{2m} V_r r^2 \dot{\phi} \sin^2 \theta + \frac{\rho C_L S}{2m} V_r \frac{r \dot{r} \dot{\phi}}{V^*} \sin^2 \theta = 0 \quad (2)$$

$$\frac{d}{dt} (r^2 \dot{\theta}) - r^2(\dot{\phi} + \omega)^2 \sin \theta \cos \theta + \frac{\rho C_D S}{2m} V_r r^2 \dot{\theta} + \frac{\rho C_L S V_r}{2m} \left(\frac{r \dot{r} \dot{\theta}}{V^*} \right) - \frac{2GMJr_e^2}{r^3} \sin \theta \cos \theta = 0 \quad (3)$$

Here $V^{*2} = \dot{\phi}^2 \sin^2 \theta + \dot{\theta}^2$

See Fig. 3 for the coordinate system used.

When describing the flight path of a lifting vehicle, the so-called equilibrium glide is often used. Equilibrium glide is defined as that flight path where the sum of the forces in the vertical direction is zero. In equation form equilibrium glide may be defined by

$$C_L \frac{\rho}{2} S V_{rel}^2 + \frac{mVi^2}{r} - mg \cos \gamma - mVi \frac{d\gamma}{dt} = 0 \quad (4)$$

The resultant flight path is sensibly horizontal.*

* For $V_i = V_{rel}$, it can be shown that along an equilibrium glide line

$$\tan V = \frac{1}{L/D} \cdot \frac{(V^2 - V_s^2)}{+V_s^2 \frac{\beta r_e V^2}{2} \left(1 - \frac{V^2}{V_s^2} \right)}$$

The flight path angle is usually less than 1° and the rate of change of flight path angle is negligible. Within 1 per cent, the radius of curvature of the flight path is equal to the radius of the earth. Hence equation (4) becomes

$$C_L \frac{\rho}{2} S V_{rel}^2 + \frac{m V_i^2}{r_e} - mg = 0 \quad (5)$$

COORDINATE SYSTEM

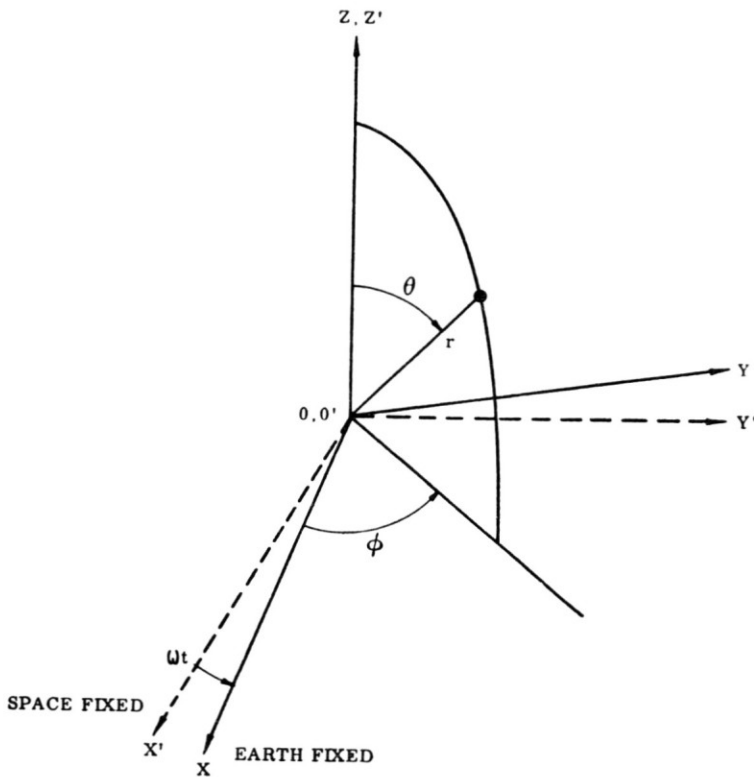


FIG. 3.

Equation (5) describes the velocity-altitude relations of a lifting vehicle in quasi-steady flight. In general, the lifting vehicle will first enter the earth's atmosphere, and then seek an equilibrium glide condition.

For velocities greater than satellite speed, negative lift is required to hold the vehicle in the atmosphere. Any equilibrium glide trajectory is inherently stable below satellite speeds, and unstable above satellite velocity. The instability, however, is akin to that of a bicycle, i.e. the rate of divergence is small.

DIRECT VS. RETROGRADE

Typical entry trajectories resulting from the lunar return mission described above are shown in Figs. 4(a) and 4(b). The aerodynamic or convective heating rates at the stagnation point were calculated by use of a modified Fay-Riddell equation⁽⁸⁾. Although the extension of this equation into the region where a high degree of ionization is expected may not be completely accurate, it is probably representative.

From Fig. 4 it can be seen that the peak heating rates are higher for retrograde entries, i.e. westward flight, than they are for the direct entry or easterly flight. The time integral of the heat rate (i.e. total heat load) is also greater for westward flight. Considering these factors as well as reduced mission time, the direct entry appears more desirable.

UNMODULATED ENTRY

Single pass entry limits for both a ballistic and a lifting vehicle are shown in Fig. 5 as a function of velocity. The entry angle is defined as a flight path angle at 400,000 feet altitude. This definition is somewhat arbitrary but is being used extensively. No lift or drag modulation was used in establishing Fig. 5. Note that the range of entry angles permissible for the ballistic vehicle is smaller and hence the re-entry corridor is considerably narrower than that for a vehicle with a lift: drag ratio of 1. As re-entry velocity increases, the corridor for both types of vehicles narrows. At escape speed, the re-entry angle tolerance for a ballistic vehicle is less than one degree and as the velocity increases the corridor for the ballistic vehicle decreases and ultimately vanishes. This occurs at about 46,600 fps.

Since 35,000 fps is a representative entry velocity, a number of re-entry trajectories have been computed for a wide variety of vehicles. These results have been presented in⁽¹⁾, and a summary results chart is shown in Fig. 6. Here the concept of virtual perigee is used. Virtual perigee is defined as that perigee that would occur if the earth had no atmosphere and were a point mass. This concept was used by Chapman⁽⁴⁾. For any re-entry trajectory, the relationship between virtual perigee and re-entry angle at any given altitude may be calculated by simple Keplerian relations. The difference in the virtual perigee at skip limit and at the 10 g limit defines the entry corridor.

LIFT: DRAG POLARS

To use Fig. 6, and for the ensuing discussion on heating and modulated entries, the variation of drag with lift must be defined. By fixing a max C_L and a max L/D , and assuming that the variation in normal force

BALLISTIC ENTRIES

ENTRY ANGLE -6°
@ 400,000 FT, 35,000 FPS
 $W/SC_D = 100 \text{ PSF}$

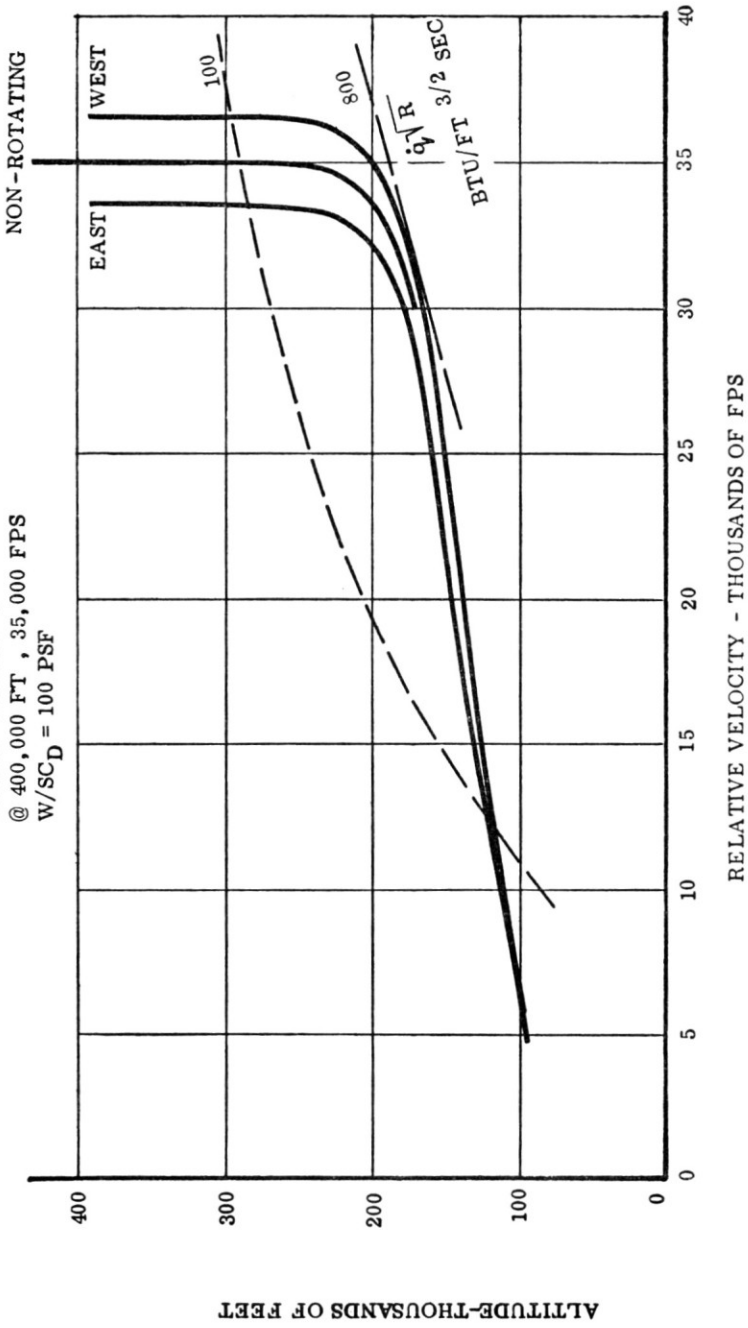


FIG. 4 (a)

LIFTING ENTRIES

ENTRY ANGLE -6°
 @ 400,000 FT., 35,000 FPS
 L/D = 1.0 W/SC_L = 100 PSF

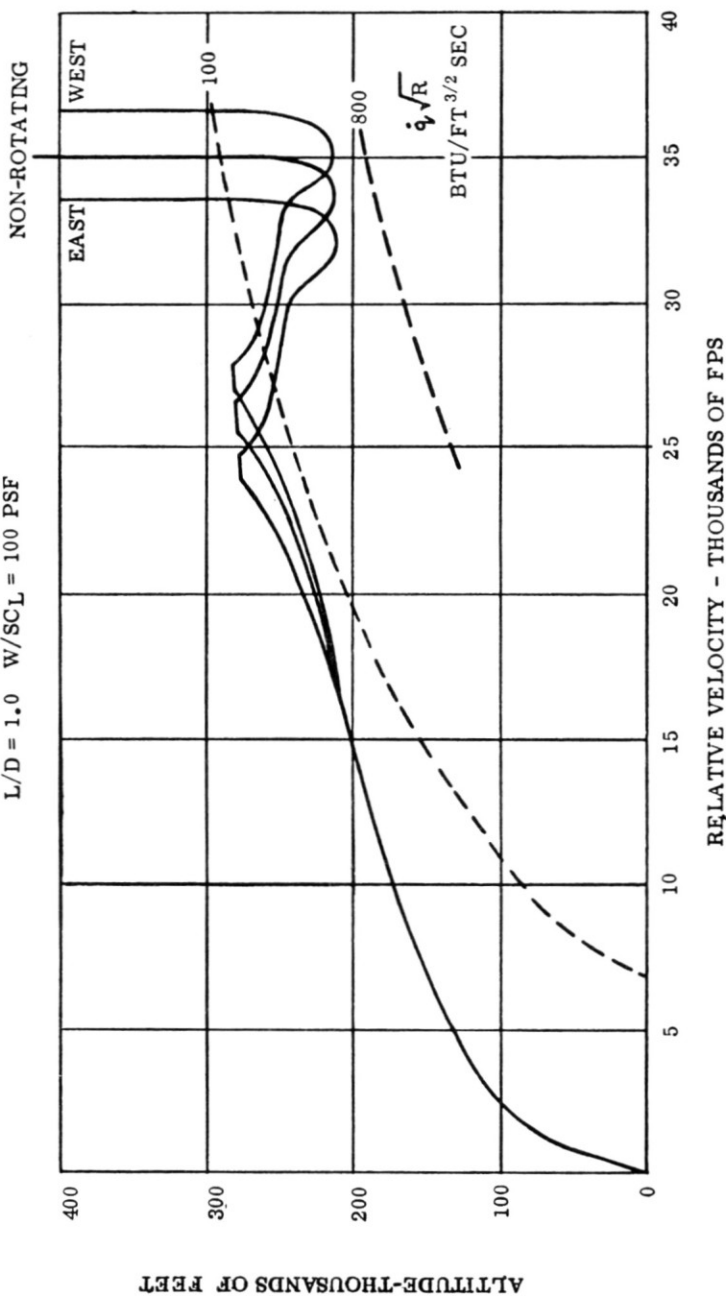
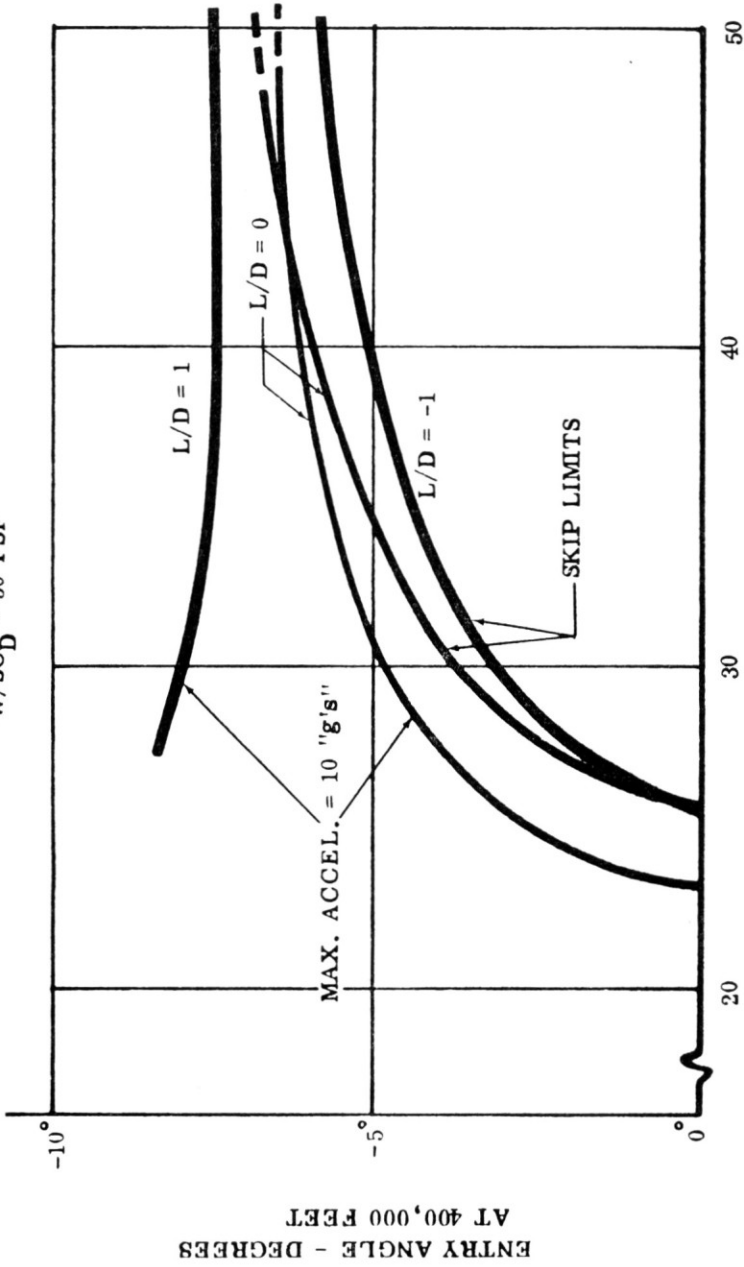


FIG. 4 (b)

TYPICAL ENTRY CORRIDORS

$W/SC_D = 50$ PSF



VELOCITY - THOUSANDS OF FPS

Fig. 5.

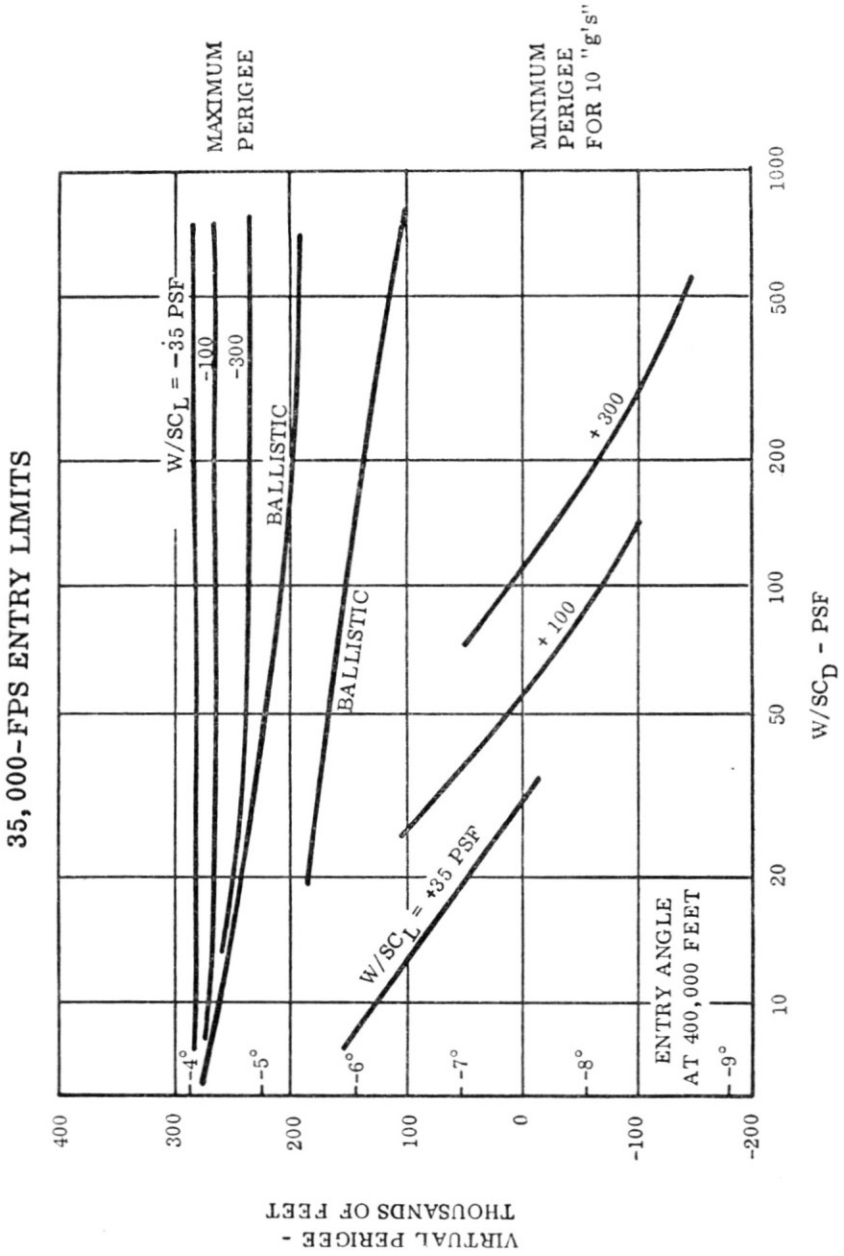


FIG. 6.

coefficient with angle of attack follows modified Newtonian theory, a complete lift: drag polar may be established. The lift coefficient can be expressed by

$$C_L = N \sin^2 \alpha \cos \alpha \quad (6)$$

In a similar fashion the drag coefficient can be expressed by

$$C_D = C_{D0} + N \sin^3 \alpha \quad (7)$$

By a suitable choice of the constants, N and C_{D0} , representative lift: drag polars were constructed.* These are shown in Fig. 7.

Since the skip limit for most lifting vehicles is dependent only on W/SC_L , for a given wing loading (W/S), the maximum skip limit occurs at max C_L , and improves with increasing max C_L . For a given wing loading, the acceleration limit in general will occur at an angle of attack below the angle of attack for max L/D . Note also that for the general case of the lifting vehicle, the two corridor limits occur at different drag coefficients, or W/SC_D 's. Hence the lifting vehicle will have an even wider corridor when provided with normal trim capability over a range of angles of attack.

RE-ENTRY HEATING

As mentioned above, the aerodynamic or convective heating rates at the stagnation point were calculated by use of a modified Fay-Riddell equation. Lower surface or flat plate heat transfer rates were calculated using the reference temperature method of Eckert⁽⁹⁾.

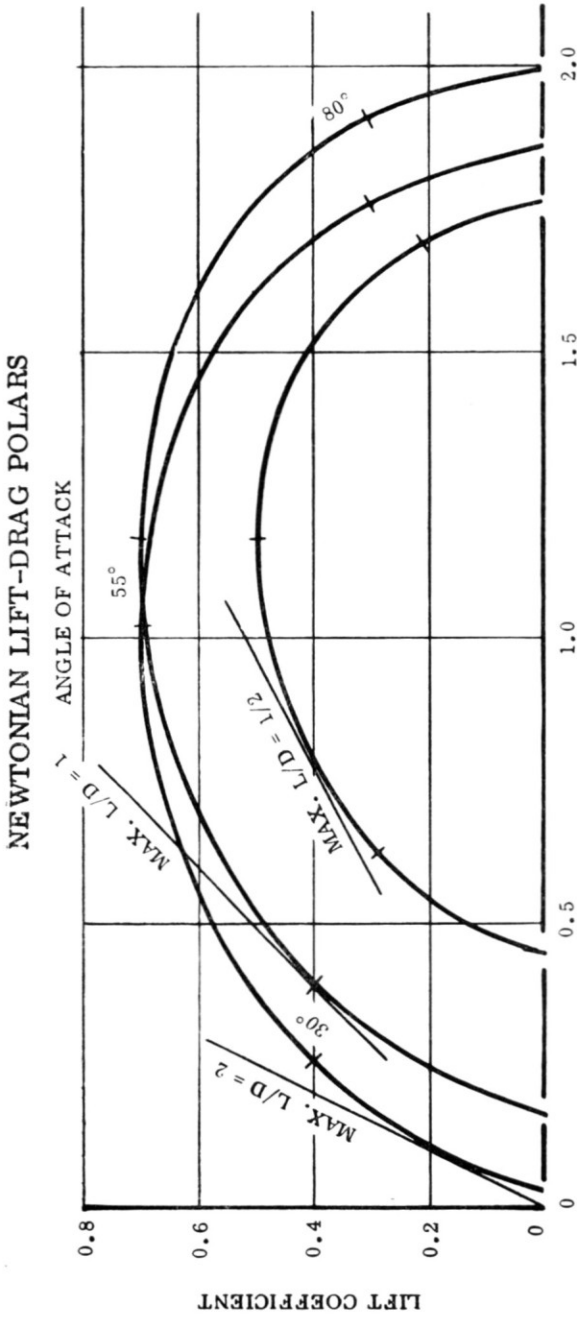
At these very high entry speeds the air behind the nose shock becomes highly ionized, and looks to the body like a hot radiator. The heat transfer due to radiation from the hot gas cap around the nose was calculated from the data of Kivel⁽¹⁰⁾. Stagnation point heating rates, both radiative and convective, are shown in Fig. 8.

Typical lifting and ballistic entry trajectories are also shown. It is apparent that, although heating rates may be relatively high, the radiative heat transfer will not contribute the major portion of the heat load.

Typical time histories of the heating rates imposed by the trajectories of Fig. 8 are shown on Fig. 9.† Note that, as would be expected, the flight time of the ballistic vehicle is considerably shorter than that of the lifting vehicle. For the lifting vehicle, the major portion of the time spent is at or close to satellite velocity. The initial entry heating pulse is roughly sinusoidal in shape with a base of about one minute duration. This type

* N is actually the stagnation pressure coefficient behind a normal shock, when used with a flat plate at high angles of attack.

† For heating histories a nose radius of 6 in. is assumed. Lower surface heat rates are calculated for a point 10 ft back from the nose.



DRAG COEFFICIENT

FIG. 7.

LIFTING AND BALLISTIC ENTRIES

ENTRY ANGLE = -6°
@ 400,000 FT, 35,000 FPS
L/D = 1.0 W/SCD = 100 PSF

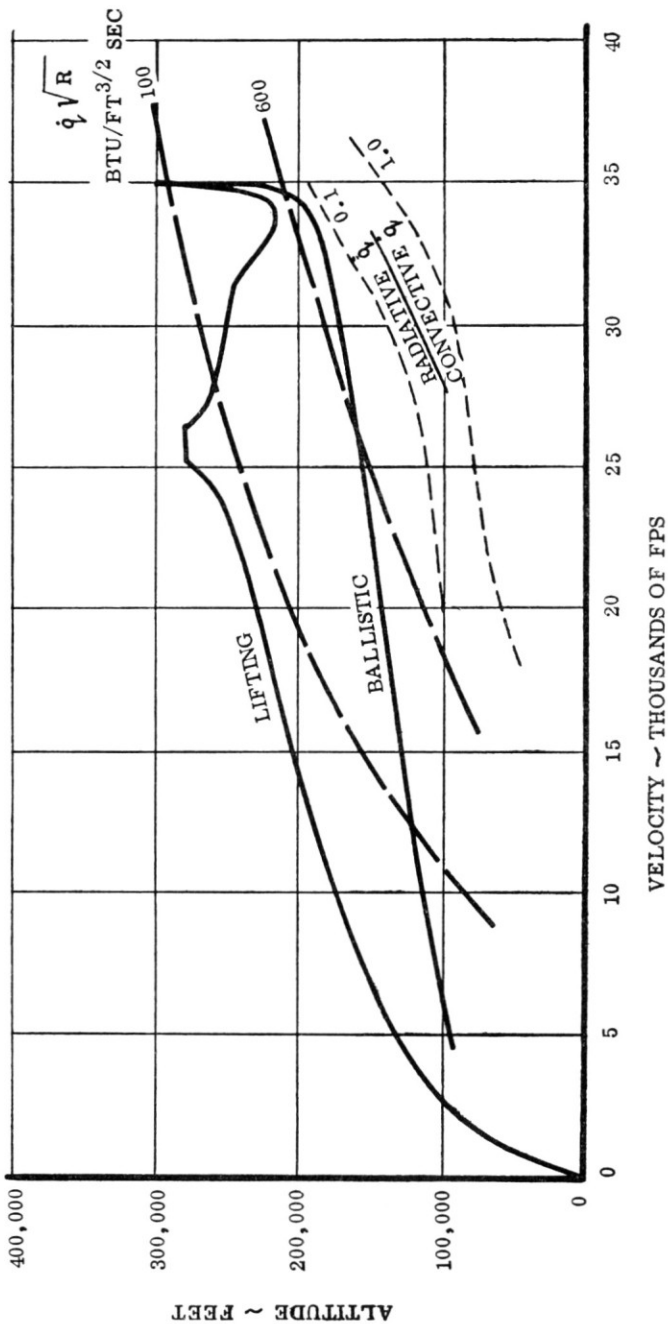


FIG. 8.

of heating pulse is typical of the kind that can be handled by ablation materials. Actually, for the ballistic vehicle, it would be desirable to use ablation materials for heat protection throughout the flight to the ground. The stagnation point heat load for ballistic vehicles entering at 35,000 fps is shown in Figure 10. Note that the total heat is sensibly proportional

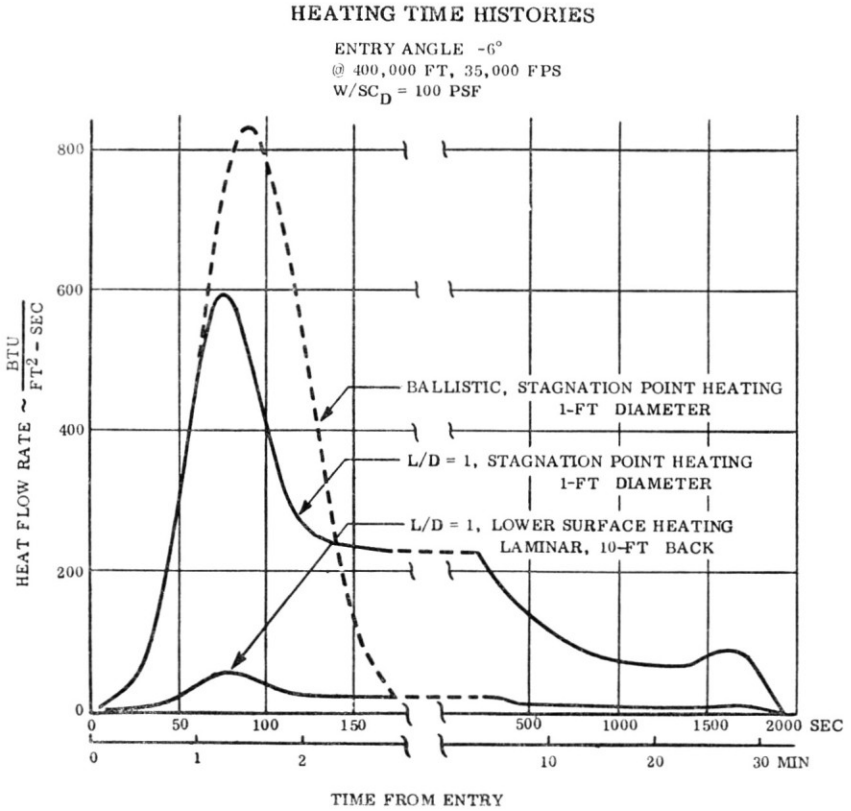


FIG. 9.

to the square root of the ballistic coefficient.* Ballistic entries made steeper than 6° would produce very high accelerations and are probably unusable for manned flight.

Certainly, in the case of the lifting vehicle, the initial pulse could be protected by ablation materials. However, below satellite speed the choice is not so obvious. Indeed, by proper selection of shape and wing loading, radiation cooling is feasible.

$$\left. \begin{array}{l}
 \text{* At skip limit } Q/S \approx 6470 \sqrt{\frac{W}{SC_D r_n}} \\
 \text{At } \nu = -6^\circ \quad Q/S \approx 5160 \sqrt{\frac{W}{SC_D r_n}}
 \end{array} \right\} V_{tfinal} \approx 0$$

From the above discussion, the total heat load, at least down to satellite velocity, is probably the important controlling design factor. Any technique that would minimize the total heat taken aboard the vehicle should also minimize the weight of heat protecting material required.

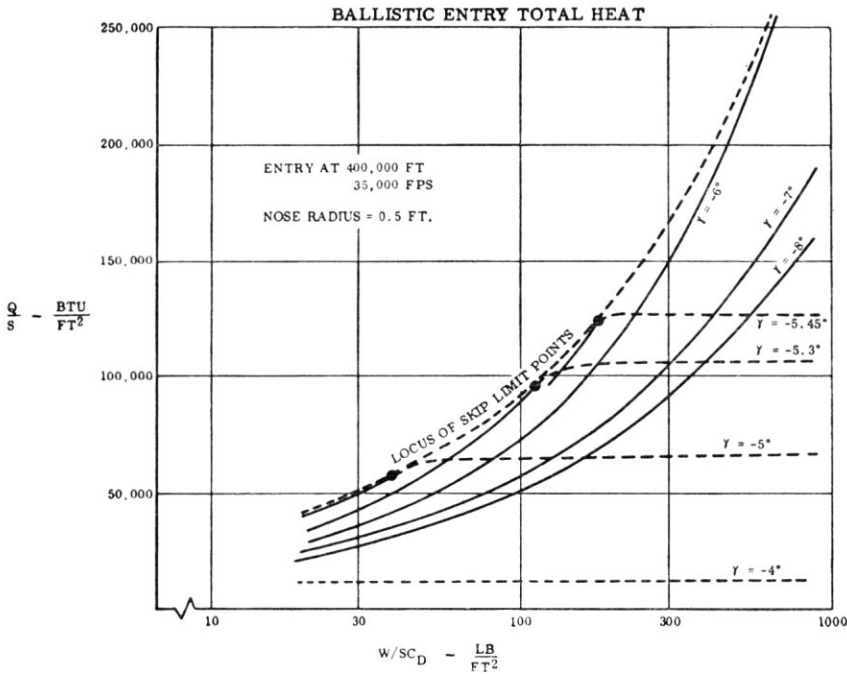


FIG. 10.

By integrating the stagnation and lower surface heat rates over the initial heating period for the lifting vehicle, a definite trade is established between the total heat taken aboard and the maximum permissible or desired acceleration. This is shown in Fig. 11. To minimize the total heat load, entry is made at a very high angle of attack, at or near maximum lift coefficient. However, to minimize the acceleration (at the expense of the total heat), entry at an angle of attack at or near maximum lift: drag ratio is better. Note that the stagnation heat load (representative of vehicle areas not sensitive to the instantaneous angle of attack but only to the trajectory) continues to increase as the entry is made at lower angles of attack.*

* This is for the initial pull-up. Along an equilibrium glide, the stagnation heat load has the general form

$$Q/S = \sqrt{\frac{W}{SC_L r_n}} \cdot \frac{L}{D}$$

The ratio of the total heat load represented by the lower surface to that represented by the stagnation region is a function of the specific design. However, desirable characteristics which can be deduced from Fig. 11 are:

1. A high maximum lift coefficient can minimize the heat load.
2. Operation at very low angles of attack does not appear beneficial.

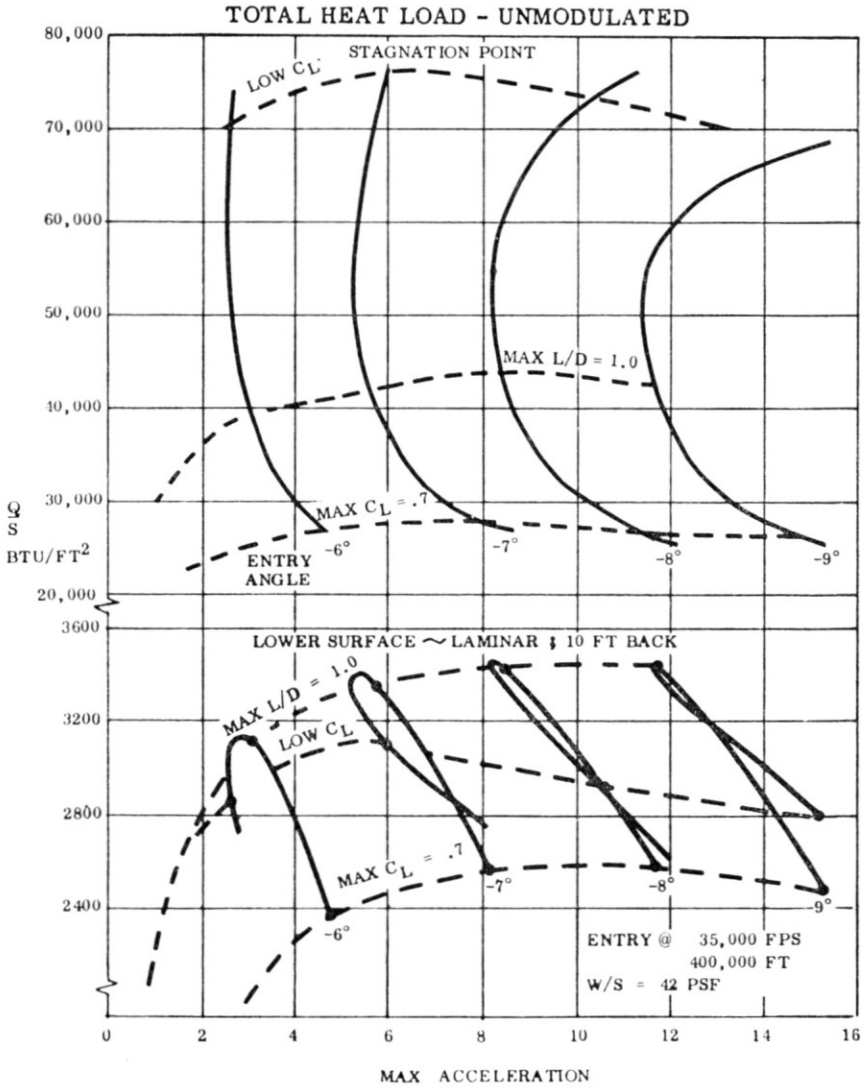


FIG. 11.

ENTRIES WITH LIFT AND DRAG MODULATION

As pointed out in refs.^(1,3,6), it is possible to widen greatly the re-entry corridor by modulating the lift and the drag. In other words, if the maximum or peak acceleration is hypothetically limited to $10 g$'s, at the onset of the $10-g$ limit during the entry trajectory, the lift or drag or both are

MODULATION EFFECT ON CORRIDOR

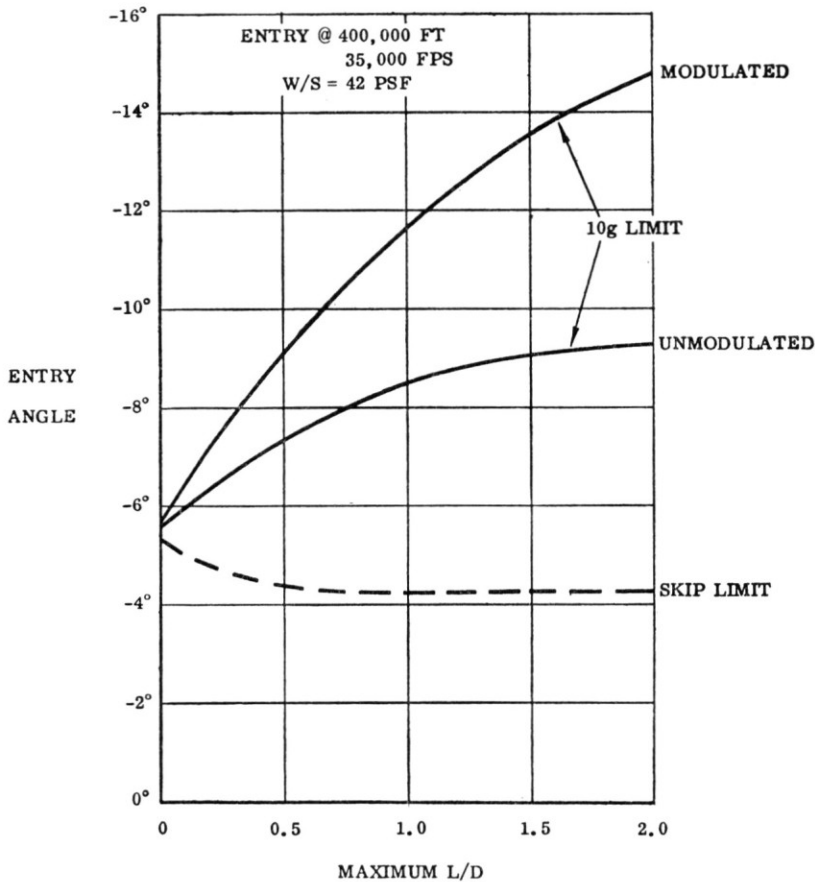


FIG. 12.

changed to keep the acceleration from exceeding $10 g$'s. Results are shown on Fig. 12. Lift:drag ratios greater than 1.0 produce little corridor improvement if no modulation is used. However, with modulation, the corridor is expanded considerably, and continues to improve with in-

creasing maximum L/D . This is because the range of modulation available (the ratio of maximum to minimum usable force coefficients) is increased.

Perhaps a more interesting correlary to the widening of the corridor is the reduction of the total heat at a given peak acceleration by using lift modulation. For example, an unmodulated entry trajectory where the

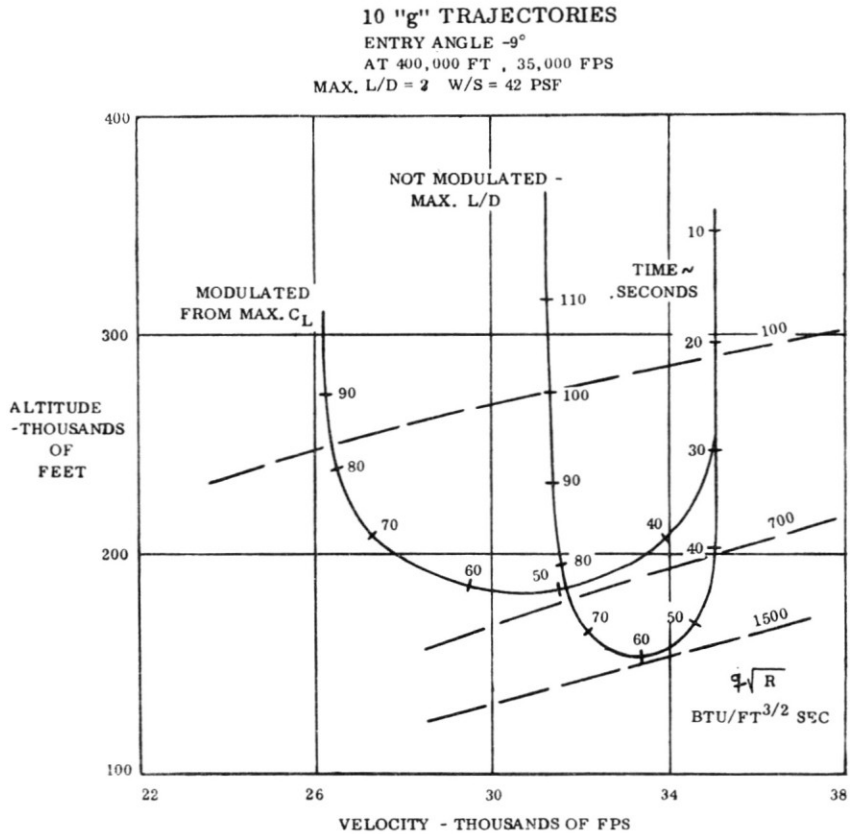


FIG. 13.

peak acceleration is 10 g 's is shown in Fig. 13. The initial flight path angle is -9° , and an L/D of 2.0 is required in conjunction with an assumed wing loading of 42 psf. This L/D was assumed to be the maximum L/D obtainable with this vehicle. By taking this same vehicle and assuming Newtonian-theory polars it is possible to start the re-entry trajectory at a much higher angle of attack at the onset of 10 g 's. This is also shown in Fig. 13. The resultant acceleration and heating histories are shown on Fig. 14.

Obviously the 10- g period must last longer if the entry is to be made in this fashion. However, there is a significant saving in both lower sur-

face and stagnation point heating loads, particularly the latter. This is shown in Fig. 15 for the case of maximum $L/D = 1.0$ and an initial flight path angle of -8° . Again a wing loading of 42 psf was assumed. The unmodulated entry data are identical to that presented in Fig. 11. All

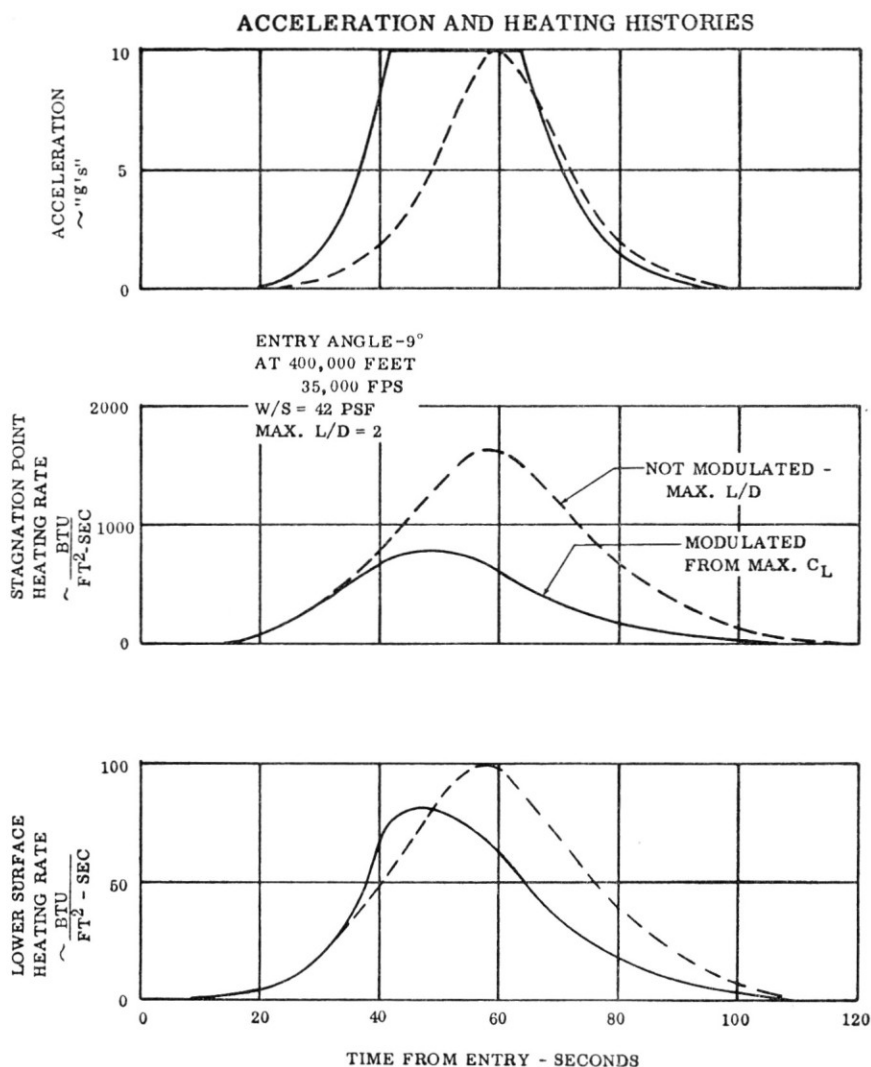


FIG. 14.

of the modulated entries are initiated at maximum C_L . Figure 15 graphically demonstrates the advantages of modulation either to reduce acceleration or total heat load or both.

For steep entries, where the acceleration and heating rates are significant, the optimum method for re-entry using lift is to begin re-entry at a very high angle of attack and the angle of attack at the onset of high accelerations. Several yardsticks for modulation are possible. One measure of magnitude of modulation is the change in angle of attack required to keep the acceleration from exceeding 10 g's. This is shown in Fig. 16, for maximum $L/D = 2$. Note that the 8° entry can be accomplished with no

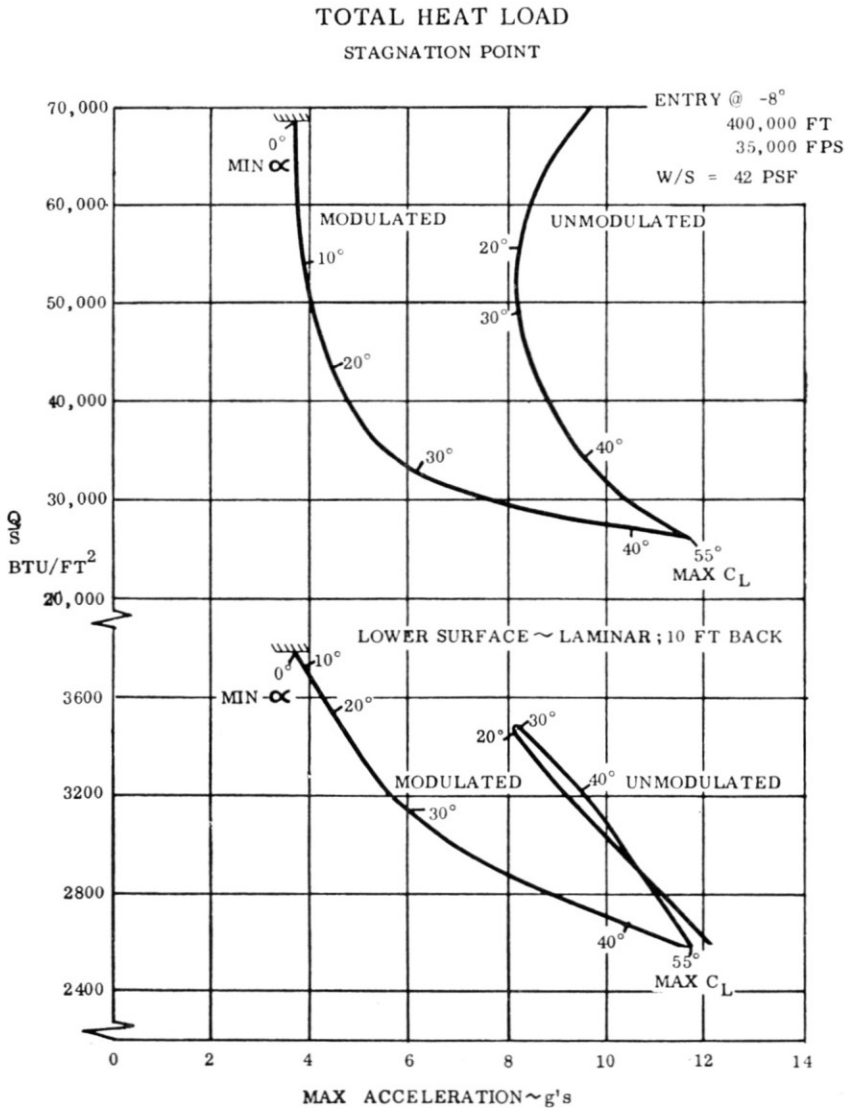


FIG. 15.

modulation if the angle of is kept relatively low. However, for the steep entries the initial angle of attack must be high (corresponding roughly to maximum C_L) if maximum acceleration is not to exceed $10 g$'s. Obviously the range of modulation available is increased by initiating entry at a high angle of attack, since modulation is always toward decreasing force coef-

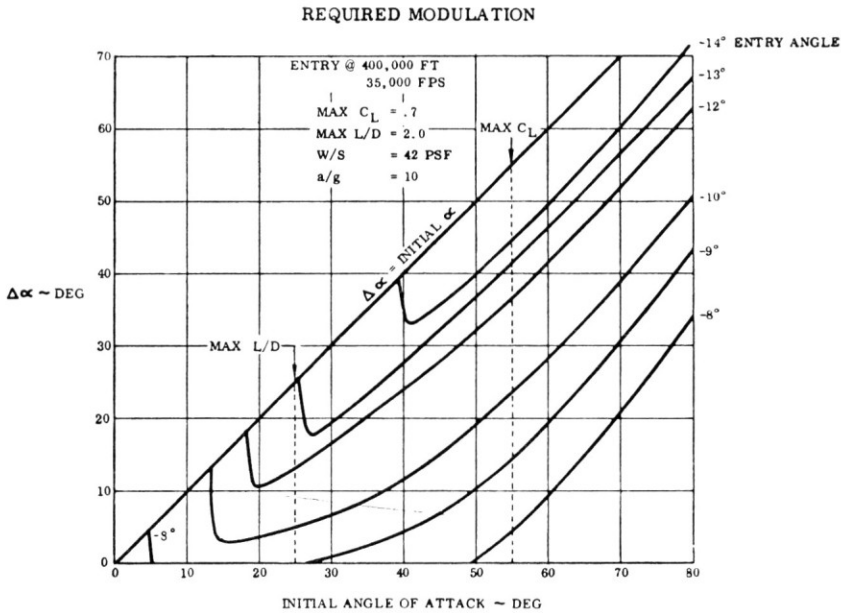


FIG. 16.

ficients. Initiating entry at 90° angle of attack, however, removed the advantage of lift deflecting the flight path away from the denser atmosphere, and is not as beneficial for corridor widening as initiating entry at about 60° angle of attack.

Heretofore the total heat only during the re-entry pull-up has been considered. The vehicle will be subject to additional heating during the flight. Entries made using modulation in general decelerate to much lower velocities during the pull-up than when no modulation is used. A much larger portion of the total energy thereby is dissipated, with accompanying savings in the heat protection required.

For example, the heating rates were integrated through the pull-up and, where required, on an equilibrium glide down to a representative velocity; namely, 27,500 ft/sec. The results are shown in Fig. 17. If the entry conditions are such that the vehicle does not decelerate to close to satellite speed during the pull-up, considerably more heat is taken on board.

TOTAL HEAT - ENTRY AT MAX C_L

$$\text{MAX } L/D = 1 \quad \text{MAX } C_L = 0.7$$

$$W/S = 42 \text{ PSF}$$

$$V_{\text{final}} \cong 27,500 \text{ FPS}$$

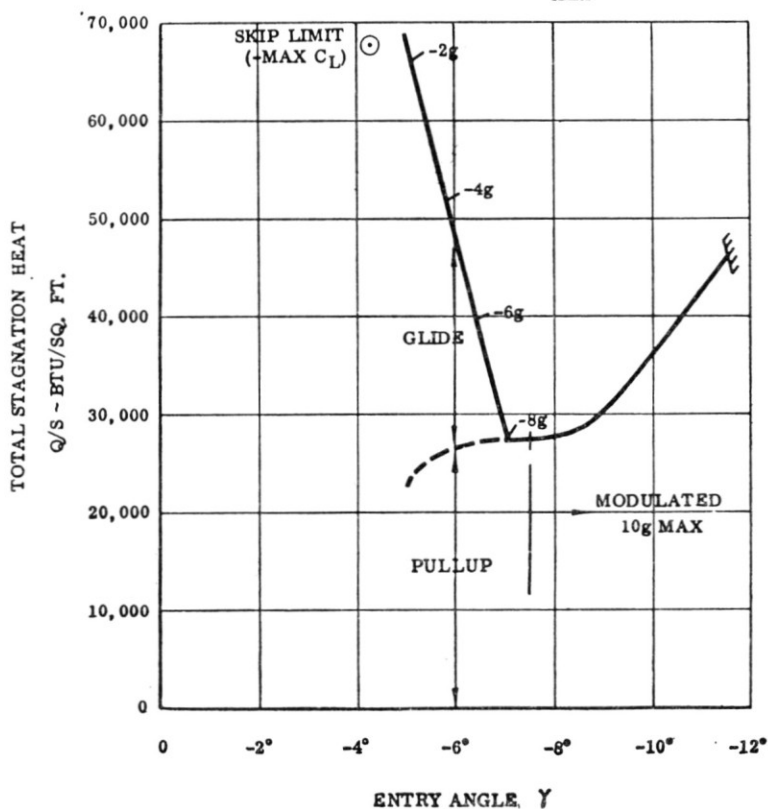


FIG. 17.

CONCLUSIONS

The return of a manned space vehicle to the earth from a lunar mission represents a formidable design task. The guidance and control requirements are probably more stringent than those required by an ICBM. Lifting entries appear attractive to reduce accelerations and provide a wider entry corridor. Modulating the lift and drag can further widen the corridor. Alternatively, for a given corridor and acceleration limit, the heat load may be reduced by modulation. Deceleration to satellite speed during the re-entry pull-up reduces the total heat for a lifting vehicle. A high maximum lift coefficient raises the skip limit and provides a wider

base for modulation for steep entries. Entries should always be started at or near maximum lift. Reducing the lift (and drag) when a limiting acceleration is reached takes maximum advantage of inherent vehicle characteristics for manned return from space.

LIST OF SYMBOLS

<i>Symbol</i>	<i>Explanation</i>	<i>Units</i>
C_D	Drag coefficient $\equiv \frac{D}{qS}$	
C_L	Lift coefficient $\equiv \frac{L}{qS}$	
D	Aerodynamic drag	pounds
g	Local acceleration of gravity	ft/sec ²
G	Gravitation constant = 3.43438×10^{-8}	lb ft ² /slug
J	Dimensionless constant = 1.637×10^{-3} (oblateness term)	
L	Aerodynamic lift	pounds
m	Vehicle mass	slugs
M	Mass of the earth = 4.09288×10^{23}	slugs
N	Parameter in modified Newtonian lift and drag coeff.	
q	Dynamic pressure $\equiv \frac{\rho}{2} V_{rel}^2$	psf
\dot{q}	Local heat rate	BTU's/ft/sec ²
Q	Total heat $\equiv \int \int \dot{q} \, dS \, dt$	BTU's
r	Radius	ft
S	Reference area	ft ²
t	Time	seconds
V	Velocity	ft/sec
W	Vehicle weight	pounds
α	Angle of attack	
β	Atmospheric density decay parameter	ft ⁻¹
γ	Flight path angle—measured with respect to the local horizontal	
Θ	Complement of the latitude	
ρ	Air density	slugs/ft ³
φ	Longitude	
ω	Angular rate of earth's rotation	radians/sec

Subscripts

<i>e</i>	Earth
<i>i</i>	Inertial
<i>n</i>	Nose
<i>o</i>	Condition at zero lift
<i>r</i>	Radial
rel	Relative
<i>s</i>	Circular satellite

REFERENCES

1. HILDEBRAND, R. B., Manned Re-Entry at Super-Satellite Speeds. IAS Preprint 60-83 Presented at the IAS National Summer Meeting, June 1960
2. ALLEN, H. JULIAN and EGGERS, A. J. Jr., A Study of the Motion and Aerodynamic Heating of Ballistic Missiles Entering the Earth's Atmosphere. NACA Report 1381 (1958)
3. LEES, LESTER, HARTWIG, F. W. and COHEN, CLARENCE B., Use of Aerodynamic Lift During Entry Into the Earth's Atmosphere. *Jet Propulsion*, Vol. 29, No. 9 (Sept. 1959) p. 633
4. CHAPMAN, DEAN R., An Analysis of the Corridor and Guidance Requirements, for Super-Circular Entry into Planetary Atmospheres. NASA TR R-55 (1959)
5. CHAPMAN DEAN R., An Approximate Analytical Method for Studying Entry into Planetary Atmospheres. NASA TR R-11 (1959)
6. GRANT, FREDERICK C., Analysis of Low-Acceleration Lifting Entry from Escape Speed. NASA TN D-249 (June 1960)
7. MINZNER, R. A., CHAMPION, R. W. and POND, H., ARDC Model Atmosphere. 1959. *Geophysics Research Directorate*, ARDC, USAF, 1959
8. FAY and RIDDELL, Stagnation Point Heat Transfer in Dissociated Air. AVCO Research Laboratory, Report No. 1, April 1957. Also, *Journal of the Aeronautical Sciences*, Vol. 25, No. 2, Feb. 1958, pp. 73-85.
9. ECKERT, E. R. G., Survey of Heat Transfer at High Speeds. WADC Technical Report No. 54-70, Wright Air Developmental Center, Wright-Patterson Air Force Base, Ohio, 1954.
10. KIVEL, BENNETT, Radiation From Hot Air and Stagnation Heating. AVCO Research Laboratory, Research Report No. 79, October 1959.

DISCUSSION

W. F. HILTON: While I appreciate the necessity of not passing 3 times through the van Allen radiation belt, where once might suffice, would it not have been better to break off the re-entry from a lunar mission and to execute one elliptical orbit? This would be accomplished by the pilot rolling the vehicle at constant high angle of incidence, turning the negative lift into positive, and breaking off the re-entry. Apogee could be held to say 700 miles, i.e. below the radiation belt, and this would give a 100 minutes cooling period, after dissipating half the total kinetic energy.

Re-entry from outer space is similar to aircraft take-off, in that both start at zero value of $Q = \frac{1}{2} \rho V^2$, and both have to use high angle of incidence to generate a small lift force, at a high value of C_L .

R. B. HILDEBRAND: The maneuver Dr. Hilton suggests is possible, and may be advantageous to a vehicle with the heat sink type of protection. The advantage is probably lost if the vehicle uses other types of heat protection schemes where intermediate cooling is of no benefit, and the increase in mission complexity deleterious.

A. BUSEMANN: There seems to be a discrepancy between the fact, that on one hand lift is so much better than the ballistic entry without lift, while there is such a large improvement by "modulation" beyond the case of "maximum L/D ". I believe the reason of maximum L/D is applied without any sense of humor. If L/D appears at, say, 10° angle of attack, then practically the same value L/D holds between 7° and 14° angle of attack just as well. It only takes half a minute to see whether 7° or 14° is much superior to the humorless angle of 10° or the difference can be seen with a short slide rule computation of 10% accuracy. This optimized angle of attack for maximum L/D should be much harder to beat by modulation.

R. B. HILDEBRAND: The remarkable improvement with modulation occurs principally because of the variation in force coefficient possible. Maximum benefit is derived by modulating from about max C_L (low L/D) to about max L/D . Further discussion of this point may be found in the reference by Grant.

H. H. PEARCEY: Mr. Hildebrand has told us that the optimum re-entry corridors will be obtained with variable L/D and with angles of attack ranging downwards from that appropriate for $C_{L_{max}}$. I am wondering whether, arising from this large range of conditions, there are a number of questions that neither Mr. Hildebrand's computer nor Dr. Busemann's slide rule can answer. I suspect that the theoreticians will, as frequently in the past, need to turn to experiment. Thus we have been shown, on the one hand, calculated lift-drag polars and, on the other, calculated heating effects. Do we know whether the assumptions made in these two sets of calculations are mutually compatible and, whether both are compatible with the type of flow that is likely to be encountered throughout the angle-of-attack range envisaged?

R. B. HILDEBRAND: While there has been very little theoretical and practically no experimental work done on heat transfer at escape speed, probably the more important consideration is whether the formulae developed for lower hypersonic speeds are appropriate in a relative sense. The data may be incorrect in an absolute sense, but the variation of heat transfer with angle of attack (which is important to this work) is probably sufficiently accurate to lead to valid conclusions. The assumptions made in the pressure force and heating calculations are mutually compatible.